1. Details of Module and its structure

Module Detail	
Subject Name	Physics
Course Name	Physics 01 (Physics Part-1, Class XI)
Module Name/Title	Unit 4, Module 7,Collision in two dimension Chapter 6, Work, Energy and Power
Module Id	Keph_10607_eContent
Pre-requisites	Kinematics, laws of motion, basic vector algebra, conservation of momentum, collision elastic and inelastic collision
Objectives	**After going through this module, the learners will be able to :** • Recall Collision in one dimension • Recognize and relate to Collision in two dimensions • Derive expressions for velocities of colliding objects after collision • Visualize Collision in 3 dimension • Solve Numerical examples
Keywords	Collision, elastic collision. collision in one dimension, inelastic collision, collision in two dimension, collision problems

2. Development Team

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TABLE OF CONTENTS

1. UNIT SYLLABUS

UNIT IV: CHAPTER 6: WORK ENERGY AND POWER

Work done by a constant force and a variable force; kinetic energy; work- energy theorem and power; Notion of potential energy, potential energy of a spring conservative and non-conservative forces, conservation of mechanical energy (kinetic and potential energies) non-conservative forces, motion in a vertical circle; Elastic and inelastic collisions in one and two dimensions.

2. MODULE-WISE DISTRIBUTION OF UNIT SYLLABUS 7 Modules

The above unit is divided into 7 modules for better understanding.

Module 1	• Meaning of work in the physical sense • Constant force over variable displacement • variable force for constant displacement • Calculating work • Unit of work • Dot product • Numerical
Module 2	• Different forms of energy • Kinetic energy • Work energy theorem • Power
Module 3	• Potential energy • Potential energy due to position • Conservative and non-conservative forces • Calculation of potential energy

Module 4	• Potential energy • Elastic Potential energy • Springs • Spring constant • Problems
Module 5	• Motion in a vertical circle • Applications of work energy theorem • Solving problems using work power energy
Module 6	• Collisions • Idealism in Collision in one dimension • Elastic and inelastic collision • Derivation
Module 7	• Collision in two dimension • Problems

Module 7

3. WORDS YOU MUST KNOW

Let us keep the following concepts in mind

- **Rigid body:** An object for which individual particles continue to be at the same separation over a period of time.
- **Point object**: **Point object** is an expression used in kinematics: it is an **object** whose dimensions are ignored or neglected while considering its motion.
- **Distance travelled**: change in position of an object is measured as the distance the object moves from its starting position to its final position. Its SI unit is m and it can be zero or positive.
- **Displacement**: a **displacement** is a vector whose length is the shortest distance from the initial to the final position of an object undergoing motion. . Its SI unit is m and it can be zero, positive or negative.
- **Speed**: Rate of change of position .Its SI unit is ms^{-1}.
- **Average speed=**: $\frac{\text{total path length travelled by the object}}{\text{total time interval for the motion}}$

  Its SI unit is ms^{-1}.

- **Velocity (v)**: Rate of change of position in a particular direction.
  Its SI unit is ms^{-1}.
- **Instantaneous velocity**: velocity at any instant of time.

$$v_{instaneous} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

  Instantaneous velocity is the **velocity** of an object in motion at a specific time. This is determined by considering the time interval for displacement as small as possible .the instantaneous velocity itself may be any value .If an object has a constant **velocity** over a period of time, its average and **instantaneous velocities** will be the same.
- **Uniform motion**: a body is said to be in uniform motion if it covers equal distance in equal intervals of time

- **Non uniform motion**: a body is said to be in non- uniform motion if it covers unequal distance in equal intervals of time or if it covers equal distances in unequal intervals of time
- **Acceleration (a)**: time rate of change of velocity and its SI unit is ms^{-2}. Velocity may change due to change in its magnitude or change in its direction or change in both magnitude and direction.
- **Constant acceleration**: Acceleration which remains constant throughout a considered motion of an object
- **Momentum (p):** The impact capacity of a moving body. It depends on both mass of the body and its velocity. Given as p = mv and its unit is kg ms^{-1}.
- **Force** (F): Something that changes the state of rest or uniform motion of a body. SI Unit of force is Newton (N). It is a vector, because it has both magnitude, which tells us the strength or magnitude of the force and direction. Force can change the shape of the body.
- **Constant force**: A force for which both magnitude and direction remain the same with passage of time
- **Variable force**: A force for which either magnitude or direction or both change with passage of time

- **External unbalanced force**: A single force or a resultant of many forces that act externally on an object.
- **Dimensional formula**: An expression which shows how and in which way the fundamental quantities like, mass (M), length (L) and time (T) are connected
- **Kinematics:** Study of motion of objects without involving the cause of motion.
- **Dynamics**: Study of motion of objects along with the cause of motion.
- **Vector:** A physical quantity that has both magnitude and direction .displacement, force, acceleration are examples of vectors.
- Vector algebra: Mathematical rules of adding, subtracting and multiplying vectors.
- Resolution of vectors: The process of splitting a vector into various parts or components. These parts of a vector may act in different directions. A vector can be resolved in three mutually perpendicular directions. Together they produce the same effect as the original vector.
- Dot product: If the product of two vectors (A and B) is a scalar quantity. Dot product of vector A and B: A.B= $|A||B|cos\theta$ where θ is the angle between the two vectors

  Since Dot product is a scalar quantity it has no direction. It can also be taken as the product of magnitude of A and the component of B along A or product of B and component of A along B.

- Work: Work is said to be done by an external force acting on a body if it produces displacement W= F.S cosθ, where work is the dot product of vector F(force) and Vector S (displacement) and θ is the angle between them . Its unit is joule and dimensional formula is ML^2T^{-2}. It can also be stated as the product of component of the force in the direction of displacement and the magnitude of displacement. Work can be done by constant or variable force and work can be zero, positive or negative.

- Energy: The ability of a body to do work

- Kinetic Energy: The energy possessed by a body due to its motion = ½ mv^2, where 'm' is the mass of the body and 'v' is the velocity of the body at the instant its kinetic energy is being calculated.

- Work Energy theorem: Relates work done on a body to the change in mechanical energy of a body i.e.,

  $W = F.S = \frac{1}{2} mV_f^2 - \frac{1}{2}mV_i^2$

- Conservative force: A force is said to be conservative if the work done by the force in displacing a body from one point to another is independent of the path followed by the particle and depends on the end points. Example: gravitational force.
- Non- conservative forces: If the amount of work done in moving an object against a force from one point to another depends on the path along which the body moves, then such a force is called a non-conservative force. Example: friction.
- Conservation of mechanical energy: Mechanical energy is conserved if work done is by conservative forces.
- Potential energy due to position: Work done in raising the object of mass m to a particular height (distance less than radius of the earth) = m g h.

- Collision: Sudden interaction between two or more objects. We are only considering two body collisions.
- **Collision in one dimension**: Interacting bodies move along the same straight path before and after collision.
- **Elastic collision**: Collision in which both momentum and kinetic energy is conserved.
- **Inelastic collision**: Momentum is conserved but kinetic energy is not conserved.
- **Coefficient of restitution**: The ratio of relative velocity after the collision and relative velocity before collision. Its value ranges from 0-1.

4. INTRODUCTION

We have considered collision in one dimension in our previous module.

What is exactly is meant by collision in two dimensions. Well if you look at the fig. while playing carom by the striker.

https://www.youtube.com/watch?v=cUUzWtFS2dw

Game of carom

https://www.youtube.com/watch?v=u5P9pjuI4xk

5. ELASTIC COLLISION IN TWO DIMENSION

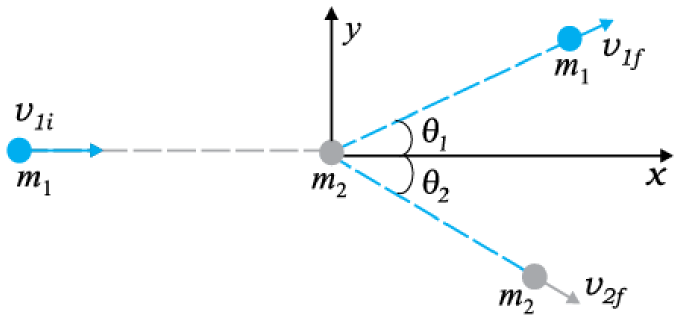
Consider two objects of masses m_1 and m_2 .The particle of mass m_1 is moving with speed u_1, we can consider object of mass m_2 to be at rest. No loss of generality is involved in making such a selection. In this situation, the object of mass m_1 collides with the stationary object of mass m_2 and this is depicted in Fig.1

In general collisions occur in two dimensions where the initial and final velocities may lie in a plane like the carom coins stayed on the plane board, and all the activity took place in two dimensions.

Think about this

- Would the football and the foot in a football game be collision in two dimension
- Would marbles in a game of marble be collisions in two dimension

Let us assume that the two objects are spherical, small and rigid we will only consider have elastic collision for simplicity.



Notice: from our earlier notation, to represent initial and final velocities as u and v.

Initial velocity of $m_2 = 0$

And for $m_1 = v_{1i} = u_1$

The final velocities for $m_1 = v_{1f} = v_1$

Final velocity of $m_2 = v_{2f} = v_2$

Linear momentum is conserved in such a collision.

Since momentum is a vector this implies three equations for the three directions {x, y, z}. Consider the plane determined by the final velocity directions of m_1 and m_2 and choose it to be the x-y plane.

The conservation of the linear momentum implies that the entire collision is in the x-y plane. The x- and y-component equations

As linear momentum is conserved in elastic collision

Along x axis:

$$m_1 v_1 \cos\theta_1 + m_2 v_2 \cos\theta_2 = m_1 u_1 + m_2 u_2$$

Along y axis it would be:

$$m_1 v_1 \sin\theta_1 - m_2 v_2 \sin\theta_2 = 0$$

Note there are three equations and four variables.

So

We must know at least one of the variables to find the other three using the three equations.

As the collision is elastic,

Total kinetic energy before collision = total kinetic energy after collision

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

In Real life:

Collisions in real life are very different from what we have studied, but the basics remain the same and we can always look for specific solutions, a lot many measurements will be needed as the ideal situation we have done for elastic collision may not be enough.

REMEMBER:

- Conservation of linear momentum can be applied only to collisions
- Or we can say conservation of linear momentum is applied whenever external force is zero
- Proper sign must be applied to indicate direction of velocities
- The velocities of colliding bodies exchange after collision only if the collision is elastic and in one dimension
- Kinetic energy is a scalar quantity and hence its components cannot be taken for collision in two dimension

EXAMPLE:

A particle of mass 2m explodes at rest explodes into two fragments of equal masses 'm'. The fragments move at right angles to each other. If the speed of one that moves at an angle of 30^0 is 4m/s, find the speed of the other.

SOLUTION:

$2mU_1 = 0$ along the horizontal and no component in the vertical direction

$mV_1 \cos 30^0 + mV_2 \cos (90^0 - 30^0) = 2mU_1 = 0$

$mV_2 \cos (90^0 - 30^0) = - mV_1 \cos 30^0 = m V_1 \sin 30^0$

$mV_1 \sin 30^0 = mV_2 \sin (90^0 - 30^0)$

$V_1 = 4$ m/s

So that, $V_2 = 4\sqrt{3}$ m/s

EXAMPLE:

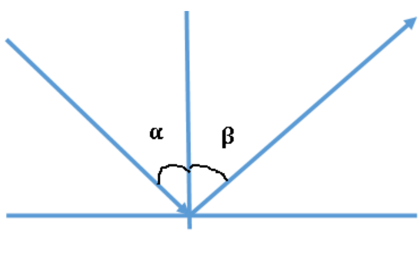
A ball of mass m hits a floor with a speed of v making an angle α with the normal, find the speed of the reflected ball and the angle of reflection if:

i) Coefficient of restitution is e

ii) Collision is perfectly elastic (e =1)

SOLUTION:

Let the angle of reflection = β



Let us write equations for momentum conservation

$$mv\ sin\beta = mu\ sin\alpha$$

$$v\ sin\beta\ = u\ sin\alpha \text{ ----------} \qquad (1)$$

$$e = \frac{vcos\beta}{ucos\alpha}$$

Or

$$v\ cos\beta\ = e\ \times u\ cos\alpha \text{ ----------} \qquad (2)$$

Squaring and adding eq. (1) and (2):

$$v^2[(\sin\beta)^2 + (\cos\beta)^2] = u^2(\sin\alpha)^2 + e^2u^2(\cos\alpha)^2$$

$$v = u\sqrt{(\sin\alpha)^2 + e^2(\cos\alpha)^2}$$

$$tan\beta\ = \frac{tan\alpha}{e}$$

Now for elastic collision e =1,

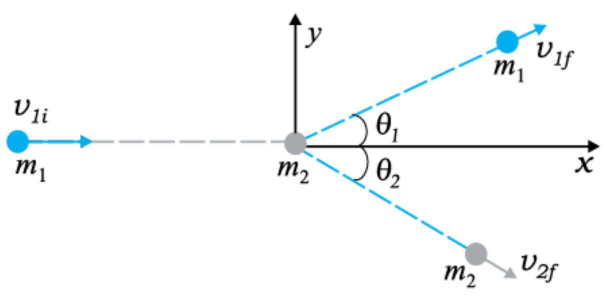
So that:

V = u and angle of reflection = angle of incidence

EXAMPLE:

Consider the collision depicted in to be between two billiard balls with equal masses m_1 = m_2. The first ball is called the cue while the second ball is called the target. The billiard player wants to 'sink' the target ball in a corner pocket, which is at an angle θ_2 = 37^0. Assume that the collision is elastic and that friction and rotational motion are not important. Obtain θ.

SOLUTION:



From momentum conservation since the masses are equal

$$v_{1i} = v_{1f} + v_{2f}$$

$$or\ {v_{1i}}^2 = (v_{1f} + v_{2f}).(v_{1f} + v_{2f})$$

$$= {v_{1f}}^2 + {v_{2f}}^2 + 2\, v_{1f} v_{2f}$$

$$= {v_{1f}}^2 + {v_{2f}}^2 + 2\, v_{1f} v_{2f} \cos(\theta_1 + 37^0) \quad \text{------(1)}$$

Since, the collision is elastic and $m_1 = m_2$ it follows from conservation of kinetic energy that:

${v_{1i}}^2 = {v_{1f}}^2 + {v_{2f}}^2$ -------(2)

By Comparing Eq. (1) and (2) we get:

$\cos(\theta_1 + 37°) = 0$

Or $\theta_1 + 37° = 90°$

Thus, $\theta_1 = 53°$

This proves the following result:

When two equal masses undergo a glancing elastic collision with one of them at rest, after the collision, they will move at right angles to each other.

The matter simplifies greatly if we consider spherical objects of equal masses with smooth surfaces, and assume that collision takes place only when the bodies touch each other. This is what happens in the games of marbles, carom and billiards.

EXAMPLE:

Answer carefully, with reasons:

(a) In an elastic collision of two billiard balls, is the total kinetic energy conserved during the short time of collision of the balls (i.e. when they are in contact)?

(b) Is the total linear momentum conserved during the short time of an elastic collision of two balls?

(c) What are the answers to (a) and (b) for an inelastic collision?

(d) If the potential energy of two billiard balls depends only on the separation distance between their centers, is the collision elastic or inelastic? (Note, we are talking here of

potential energy corresponding to the force during collision, not gravitational potential energy).

SOLUTION:

a) Decrease

b) Kinetic energy

c) External force

d) Total linear momentum and total energy if the system is isolated

6. INELASTIC COLLISION IN TWO DIMENSIONS

When the collision is inelastic, we would no longer have the one equation, corresponding to the conservation of kinetic energy, available with us. In such a case, we are effectively left with only two equations, based on the law of conservation of momentum.

In general, then, we would need information about two of the four unknowns so as to able to solve this collision problem. However, in some special cases of inelastic collisions, it is possible to get sufficient-even complete information–about the speed and direction of motion after the collision. The example given below is an illustration of a special case of an inelastic collision in two dimensions.

7. SUMMARY

In this module we have learnt:

- Collision in two dimensions follows all the rules of collision in one dimension.
- Momentum of the system is conserved in elastic and inelastic collision in two dimensions.
- Kinetic energy of the interacting particles is conserved in elastic collision.
- Real life collision may take place in two or 3 dimensions.
- The velocity of interacting particles can be found by considering the collision in each of the dimensions by selecting suitable components of the velocity vector.